01 Lines

Calculator Free

1. [5 marks: 1, 2, 2]

A line passes through the points (1, 2) and (5, 22).

(a) Find the gradient of this line.

(b) Find the equation of this line.

(c) Is (3, 25) on this line? Justify your answer.

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2. [3 marks]

The points (–2, 5), (3, k) and (5, 12) are collinear. Find the value(s) of k.

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3. [3 marks]

The line $ax + by = 18$ passes through the point (1, –4) and has a gradient of 2. Find a and b.

Calculator Free

6. [7 marks: 3, 2, 2]

The lines $2x + 3y = 12$ and $4x + 5y = 20$ meet at the point P.

(a) Find the coordinates of P.

(b) Find the equation of the line through P and parallel to the line with equation $2x + y = 10$.

(c) Find the equation of the line through P and perpendicular to the line with equation $2x + y = 10$.

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7. [3 marks: 2, 1]

Consider the line $2x + by = c$ where c is a constant.

(a) Find b if this line has gradient –4.

(b) Find c if this line has an x–intercept of 6.

Calculator Assumed

12. [10 marks: 2, 1, 3, 4]

Bill, a plumber charges a call-out fee of \$100 plus \$80 per half hour or part thereof. Ian, another plumber does not charge a call-out fee but charges \$180 per hour or part thereof.

(a) How much will Bill charge for a job that is estimated to take exactly 4 hours?

(b) How much will Ian charge for a job that is estimated to take exactly 4 hours?

(c) Determine, which plumber will be cheaper to employ if a job is estimated to take 3 hours and 20 minutes. Justify your answer.

(d) Under what conditions will it be cheaper to employ Bill? Justify your answer.

02 Quadratics

Calculator Free

1. [8 marks: 3, 2, 3]

A parabola has equation $y = (x - 2)(5 - x)$.

(a) Find the coordinates of the x and y intercepts of the parabola.

(b) Find the equation of the line of symmetry.

(c) Find the coordinates of the turning point of the parabola and state the nature of the turning point.

2. [6 marks: 3, 3]

A parabola has equation $y = 10 - 6x - 3x^2$.

(a) Find the coordinates of the turning point and state its nature.

(b) Find the exact x-intercepts.

Calculator Free

8. [4 marks]

A parabola has equation $y = k(x - a)(x - b)$ where k, a and b are constants with $a < b$. Find a, b and k if the parabola has an x-intercept at $(-3, 0)$, a turning point at $(1, 32)$ and a y-intercept at $(0, 30)$.

9. [6 marks: 1, 1, 2, 2]

Consider the parabola with equation $y = f(x) = (x - 2)(x + a)$ where a is a constant.

(a) Find a if the parabola has exactly one root.

(b) Find a if $f(2) = f(4) = 0$.

(c) Find a if $f(0) = 10$.

(d) Find a if the parabola has a turning point at $x = 3$.

Calculator Free

10. [12 marks: 2, 3, 3, 2, 2]

A parabola has equation $y = f(x)$ where $f(x) = k(x + a)^2 + 16$ where a is a constant.

(a) Find a and k if the parabola has a turning point at $(1, 16)$.

(b) Find a and k if the parabola has a turning point at $(-2, 16)$ and $f(0) = -4$.

(c) Find a and k if $f(3) = f(-5) = 0$.

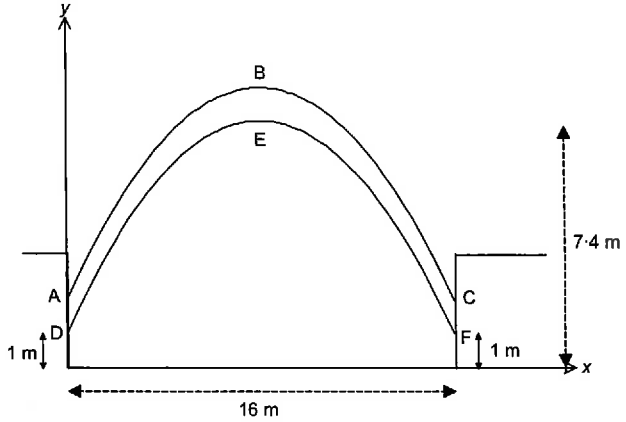
(d) Find k if the parabola has no roots.

(e) Explain clearly why the parabola cannot have exactly one root.

Calculator Assumed

16. [9 marks: 3, 1, 1, 1, 3]

The diagram below shows the cross section of an arch. ABC and DEF are the top and bottom edges of the arch respectively. Each of these edges is approximately parabolic in shape. The edges ABC and DEF are "parallel" with ABC positioned 1 metre above DEF. D and F are each 1 metre above the road level. The road level is modelled by the x-axis. The vertical line through D and A is modelled by the y-axis.



(a) Find the equation of the bottom edge of the arch (DEF).

(b) Find the equation of the top edge of the arch (ABC).

Calculator Assumed

16. (c) Find the coordinates of B, the highest point on the arch.

(d) What is the clearance of the arch (above road level) at a point which is horizontally 5 m from D?

(e) At what horizontal distance from D is the clearance of the arch 5 m above the road level?

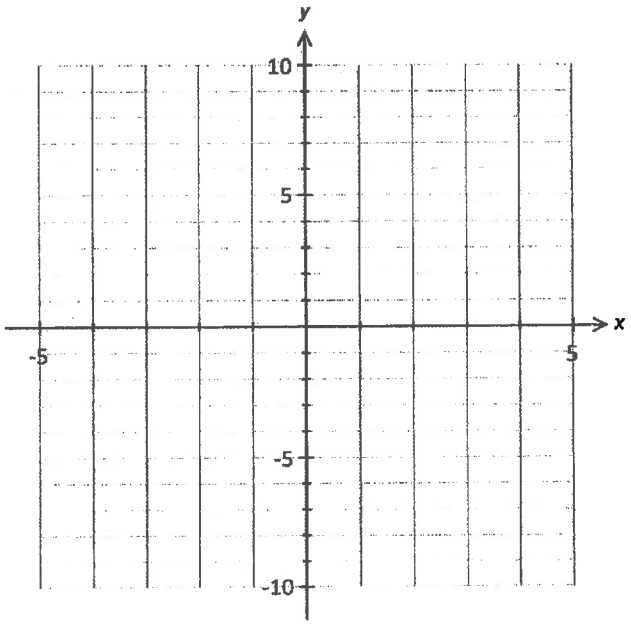
Calculator Free

2. [7 marks: 4, 3]

A curve has equation $y = 2x^3 - x^2 - 2x + 1$.

(a) Find the coordinates of the x-intercepts of this curve.

(b) Sketch this curve for $-1.5 \le x \le 2$.



Calculator Free

5. [7 marks: 1, 2, 2, 2]

Equations of cubic curves can be written in the form $y = k(x - a)(x - b)(x - c)$ or $y = k(x - a)(x - b)^2$ or $y = ax^3 + bx^2 + cx + d$. Find a possible equation of a cubic curve if this curve has:

(a) exactly three roots $x = 1, 2, -1$.

(b) exactly three roots $x = 1, 2, -1$ and y-intercept at $(0, -6)$

(c) exactly two roots $x = -1, 1$ and y-intercept at $(0, -6)$.

(d) has exactly one root $x = 1$ and y-intercept $(0, 2)$

6. [6 marks: 1, 1, 2, 1, 1]

Consider the cubic curves :

I $y = (x - 1)(x + 2)^2$ II $y = (x + 1)(x^2 - 1)$

III $y = (x - 1)^3 + 1$ IV $y = (x + 1)(1 - x)(x + 3)$

(a) Which of these curves have negative y-intercepts?

(b) Which of the given curves has three distinct (different) roots?

Calculator Free

8. [12 marks: 2, 3, 2, 5]

Consider the cubic equation $y = f(x) = k(x + 2)(x^2 - 3x + c)$ where k and c are constants.

(a) Find the value of c if $f(4) = f(-2) = f(-1) = 0$.

(b) Find the value(s) of c if the cubic curve has three roots.

(c) Find the value(s) of c if the cubic has exactly two roots.

(d) Find the values of k and c if $f(-4) = f(-2) = 0$ and $f(0) = -4$.

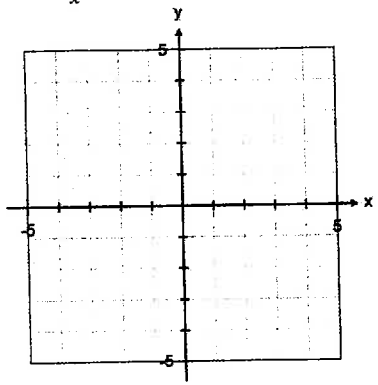
04 Rectangular Hyperbolas

Calculator Free

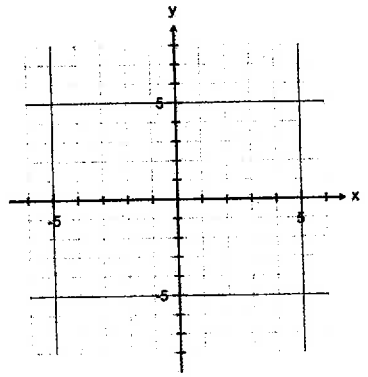
1. [4 marks: 2, 2]

Sketch in the axes provided, the graph of y against x. Show clearly all intercepts (if any) and asymptotes (if any).

(a) $y = \frac{2}{x}$ for $-4 \le x \le 4$.



(b) $y = -\frac{4}{x-1}$ for $-3 \le x \le 5$



2. [6 marks]

Complete the table below for the following curves.

Curve	x-intercept	y-intercept	Asymptote parallel to the	
			x-axis	y-axis
$y = \frac{4}{x-3}$				
$y = \frac{-3}{2x+9}$				
$y = \frac{5}{2-x}$				
$y = \frac{15}{x+5} - 3$				
$(x + 2)\, y = 10$				
$(x - 1)\,(y - 2) = 10$				

Calculator Free

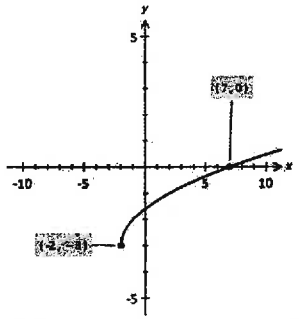
3. [4 marks: 2, 2]

(a) State two possible equations for the curve with equation $y = a + k\sqrt{x+b}$ if the curve has $x \leq 2$ and $y \leq -3$.

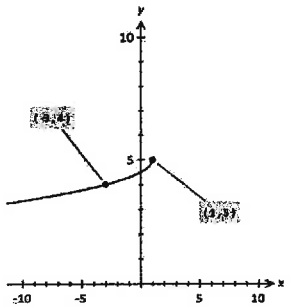
(b) State two possible equations for the curve with equation $y = a + k\sqrt{x+b}$ if the curve has $x \geq -3$ and $y \geq 5$.

4. [6 marks: 3, 3]

(a) Find the equation of the curve drawn below with equation $y = a + k\sqrt{x+b}$.



(b) Find the equation of the curve drawn below with equation $y = a + k\sqrt{x+b}$.



Calculator Free

2. [10 marks: 2, 3, 2, 3]

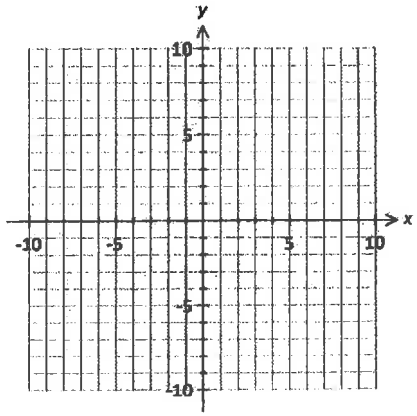
Consider the circle with equation $(x + 2)^2 + (y + 3)^2 = 25$.

(a) Find the coordinates of the x-intercepts.

(b) Find the coordinates of the y-intercepts.

(c) Determine the coordinates of the centre of this circle and its radius.

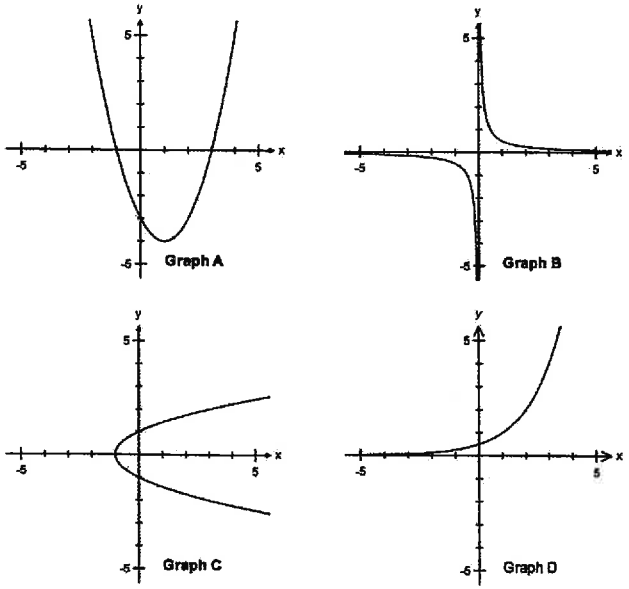
(d) On the axes provided below, sketch this circle.



Calculator Free

3. [4 marks: 1 each]

Match each of the following graphs with an equation from the given list.



Equation I: $y = \dfrac{1}{2x}$

Equation II: $y = x^2 - 2x - 3$

Equation III: $y = 2^{x-1}$

Equation IV: $y^2 = x - 1$

Equation V: $y = (x+1)^2 - 4$

Equation VI: $x = y^2 - 1$

Equation VII: $y = 2^x$

Equation VII: $y = \dfrac{1}{x}$

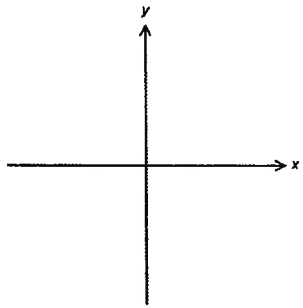
Graph	Equation
A	
B	
C	
D	

Calculator Free

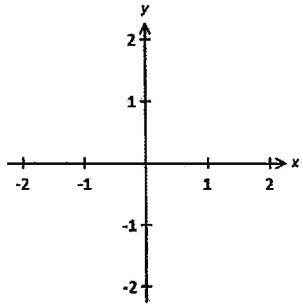
9. [12 marks: 4, 4, 4]

(a) Make a sketch of $ax + by = c$ where a, b and c are constants if:

(i) $a < 0$ and $b = 0$ and $c > 0$

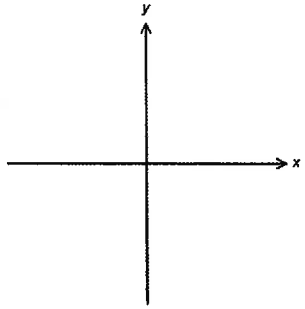


(ii) $a = b = c$

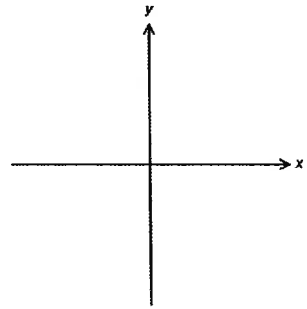


(b) Make a sketch of $y = ax^2 + bx + c$ where a, b and c are constants if:

(i) $a < 0$ and $b = 0$ and $c = 0$

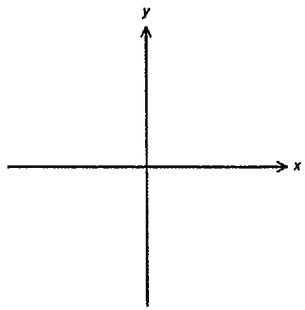


(ii) $a > 0$ and $b^2 = 4ac$

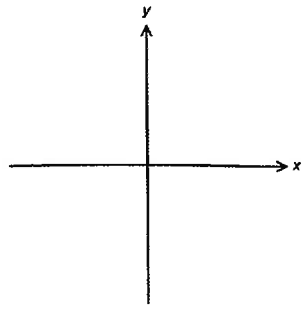


(c) Make a sketch of $y = k(x + a)(x + b)(x + c)$ where k, a, b and c are constants if:

(i) $k < 0$ and $a = b = c = 0$



(ii) $k > 0$ and $a = -b$ and $c = 0$



10 Transformations on Curves

Calculator Free

1. [10 marks: 2, 2, 2, 2, 2]

Describe a sequence of transformations required to convert $y = f(x)$ into $y = g(x)$.

(a) $f(x) = x^2$ and $g(x) = (x-2)^2 + 4$

(b) $f(x) = x^3$ and $g(x) = -(2x)^3$

(c) $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{1-x}$

(d) $f(x) = 3^x$ and $g(x) = -3^{x+1}$

(e) $f(x) = (2x+1)^2$ and $g(x) = x^2$.

2. [4 marks: 2, 2]

Describe a sequence of transformations required to transform:

(a) $x^2 + y^2 = 100$ into $(x+5)^2 + (y-6)^2 = 100$

(b) $(x-2)^2 + (y-1)^2 = 64$ into $(x+7)^2 + (y+3)^2 = 64$

Calculator Free

5. [10 marks: 2, 2, 2, 2, 2]

Identify the sequence of transformations required to map:

(a) $y = f(x)$ to $y = 2f(2x)$

(b) $y = f(x)$ to $y = f(2x+1)$

(c) $y = f(x)$ to $y = f(2(x+1))$

(d) $y = f(x)$ to $y = f(1-x)$

(e) $y = f(x)$ to $y = 1 - f(x)$

11 Equations

Calculator Free

1. [17 marks: 2, 2, 3, 3, 3, 4]

   Solve for x:

   (a) $2x - 5 = -3x + 4$

   (b) $(2x - 5)(4 - 3x) = 0$

   (c) $4x^2 - 49 = 0$

   (d) $x^2 + 1 = 4x - 3$

   (e) $(2x - 1)^2 - 25 = 0$

   (f) $x^2 + 4x - 3 = 0$

Calculator Free

2. [20 marks: 2, 2, 1, 3, 3, 5, 4]

   Solve for real values of x:

   (a) $(x - 5)(x + 3)(1 - 4x) = 0$

   (b) $(x + 3)(x^2 - 36) = 0$

   (c) $(x^2 + 1)(2x - 5) = 0$

   (d) $(x^2 - 5x + 6)(3 - 2x) = 0$

   (e) $x^3 = x^2 + 2x$

   (f) $x^3 + 4x^2 - 7x - 10 = 0$

   (g) $2x^3 + 5x^2 - 4x - 3 = 0$

Calculator Free

3. [15 marks: 3, 3, 3, 3, 3]

Solve for x:

(a) $\frac{3}{x} = x + 2$

(b) $\frac{2}{x-1} = \frac{1}{x+4}$

(c) $\frac{-1}{x+1} = x + 3$

(d) $\frac{1}{x} = x + 1$

(e) $x - 5 = \frac{1}{x--1}$

Calculator Free

5. (c) $2x + y = 10, 4x + 2y = 8$

(d) $2x + 3y = 4, 3x + y = -1$

6. [10 marks: 2, 2, 2, 2, 2]

Solve simultaneously for x and y where x and y are both integers:

(a) $x^2 + y^2 = 10, x = -1$

(b) $(x-1)^2 + (y+2)^2 = 13, y = 1$

(c) $x^2 + y^2 = 2, x + y = 0$

(d) $x^2 + y^2 = 5, x + y = 3$

(e) $x^2 + y^2 = 41, x + y = 9$